O K L A H O M A S T A T E U N I V E R S I T Y

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

ECEN/MAE 5513
Stochastic Systems
Fall 2011
Midterm Exam #1

PLEASE DO ALL FIVE PROBLEMS

Name : _____

E-Mail Address:_____

Problem 1:

A missile can be accidently launched if two relays *A* and *B* both have failed. The probabilities of *A* and *B* failing are known to be 0.01 and 0,03, respectively. It is also known that *B* is more likely to fail (probability 0.06) if *A* has failed.

a) What is the probability of an accidental missle launch?
b) What is the probability that *A* will fail if *B* has failed?
c) Are the events "*A* fails" and "*B* fails" statistically independent?

Problem 2:

Suppose the depth of water, measured in meters, behind a dam is described by an exponential random variable having a probability distribution function

$$F_X(x) = \left[1 - e^{-x/13.5}\right] u(x).$$

There is an emergency overflow at the top of the dam that prevents the depth from exceeding 40.6m. There is a pipe placed 32.0m below the overflow (ignore the pipe's finite diameter) that feeds water to a hydroelectric generator.

a) Given that water is not wasted in overflow, what is the probability the generator will have water to drive it?
b) What is the probability that water will be too low to produce power?

Problem 3:

Prove that central moments μ_n are related to moments m_k about the origin by

$$\mu_n = \sum_{k=0}^{n} \binom{n}{k} (-\overline{X})^{n-k} m_k \ .$$

Problem 4:

The probability density function of chi-square randm variable,

$$f_X(x) = \frac{x^{(N/2)-1}}{2^{N/2}\Gamma(N/2)} u(x) e^{-x/2},$$

has a characteristic function,

$$\Phi_X(\omega) = \frac{1}{(1 - j2\omega)^{N/2}}.$$

Use this function to find the mean and second moment.

Problem 5:
In a computer simulation, it is desired to transform numbers, that are values of a random variable uniformly distributed on (0,1), to numbers that are values of a *Weibull* distributed random variables, as defined by

$$F_X(x) = \left[1 - e^{-ax^b}\right]u(x)$$

with $a > 0, b > 0$. Find the required transformation.